Explorer: A Program for Common Factor Analysis and Related Models for Data Analysis

User's Guide

James S. Fleming, PhD

Southwest Psychometrics and Psychology Resources
http://swppr.org

Release 4.0

Explorer: A Program for Common
Factor Analysis and Related Models for Data Analysis

A User's Guide

Release 4.0, June, 2017

Southwest Psychometrics and Psychology Resources (http://swppr.org)

Dedication

This program is dedicated to the memory of Henry F. Kaiser, my mentor at the University of California. In addition to his many theoretical contributions to factor analysis, Kaiser contributed greatly to the practical application of exploratory factor analysis as realized in the computer program. He produced many programs throughout his career, and each of these he thought would be his last. But he continually revised and reformulated his ideas about what a good program should accomplish. Kaiser took many radical shifts in direction, and continued to do so until the very end of his life.

Explorer is a program based on my own ideas about what is useful in an exploratory factor analysis program. Kaiser and I were not always in agreement about this, but still, his influence on my thinking is very much alive in Explorer. In that same spirit I regard Explorer as an organic product, which will continue to grow and develop.

Acknowledgements

The following people have contributed to the development of Explorer through their suggestions and feedback:

Peter Bentler, Robert Dean, Pawel Grygiel, Bob Jennrich, Rod McDonald, Augustine Osman, and Sam Pinneau. I am also grateful to Barbara Tabachnick for the use of the WISC-R data, and especially, I thank my wife, Anita Fleming, for her support with this project and in all other matters.

Also, as an experienced data analyst and computer programmer (I am not a mathematician), I am indebted to all of the developers in the fields of psychometrics and statistics for their pioneering work in factor analysis.

How to Cite Explorer (APA)

Fleming, J. S. (2017). Explorer: A program for common factor analysis and related methods for data analysis, Release 4.0. [Online software and manual.] Retrieved from: http://swppr.org.

Disclaimer

Preface

(with Caveat)

This manual instructs people in the use of a program that was designed for many different kinds of exploratory data analysis. Some of these are very basic and familiar, others less so. Less familiar features include some relatively recent methods such as the exploratory bi-factor rotations and coefficient omega for factor scales, which I hope that researchers will eventually find useful. Other features, such as “Harris” component analysis, are more obscure, and will probably be less interesting to most users. I include these not just for completeness, but also because people with psychometric backgrounds may find some of them methodologically intriguing.

My caveat is simply this: the manual will not teach anyone the proper use of factor analysis and related methods, so it is assumed that readers already understand the underlying theory and necessary mathematics required for these methods. If this is not the case then are many useful textbooks on the factor analysis or multivariate analysis that can be helpful. The texts by Lee and Comrey (1992), Gorsuch (1983), Harman (1976), and Mulaik (2010), are all useful resources.

Having said that, please note that I have included several “Help Boxes” that offer some of my thoughts on how to choose a method of analysis, how to decide on the correct number of factors, interpreting matrix output, the comparative usefulness of exploratory versus confirmatory factor analysis, and methods of assessing scale reliability. I do hope that some readers may find these helpful. But they are based on my own opinions and preferences; so they are neither definitive nor are they intended as a substitute for a solid course on factor analysis.

-Jim Fleming, June, 2017

What’s New in Explorer?

Explorer 4.0 includes bootstrap standard errors for factor loadings and correlations (see Appendix A), and maximum likelihood estimation.

Contents

Appendix A

Lists of Figures, Tables, and Text Help Boxes

List of Figures — Page

List of Tables — Page

List of Text Help Boxes

Explorer: A Program for Common Factor Analysis and Related Models for Data Analysis

User's Guide

A Brief History

Explorer has a long history but a brief existence as complete package. It was designed and coded by James Fleming, beginning with some very early matrix algebra and eigenvalue/vector routines written in the 1970s, tied to a "Little Jiffy" program (principal components with varimax rotation, etc.). A few subsequent extensions were also made, but it wasn't until much later, after setting it aside several times, that the current program began to take shape. Although Explorer began with Fortran IV code, it was eventually expanded and re-written in Fortran 90, then most recently in F2003. The earliest routines ran on huge mainframes, some of which filled entire rooms; then these were converted for use on mini-computers, that occupied only a corner of a room; with the present version running on later releases of the Windows environment, fitting neatly on your lap or desktop. (Or even on your smartphone—if your eyes are better than mine.)

Overview of Capabilities

Explorer Performs Three Exploratory Models

The Explorer program performs three exploratory models for data analysis:

- Multiple common factor analysis, in the tradition of L.L. Thurstone (1947).
- Component analysis, including principal components, due to Harold Hotelling (1933).
- Image analysis, as originated by Louis Guttman (1953).

However, the emphasis here is on the common factor model.

Range of Applications

Explorer is designed as:

- A general purpose exploratory factor analysis program.
- A program with special features for constructors of multidimensional tests designed to be "factor pure," with helpful scale and item level statistics.
- A truly exploratory program with features geared toward examining data sets with complex variables (variables that saturate more than one factor) having unknown factor structure.

- A program that facilitates testing hypotheses regarding factor structure within the context of exploratory models.
- A practical program for beginners as well as a program with advanced features that will be of interest to students of psychometrics.

Factor Extraction Methods

- *Principal components/principal axis.* Common factor analysis and principal component analysis may be performed by the principal axes method; by decomposition of correlation or covariance matrix into eigenvalues and eigenvectors. Likewise, image analysis, alpha factor analysis, and image common factor analysis involve an eigenvalue/vector decomposition of the appropriate correlation or covariance matrix, as does ordinary or unweighted least-squares factor analysis.

- *MICA Extraction by pre-defining the factor structure (multiple independent cluster analysis option).* Refer to the section on confirmatory features for details.

Model and Method Varieties

- *Factor analysis:* Varieties include principal axes factor analysis, maximum likelihood (e.g., Jöreskog, 1967), ordinary least-squares (aka OLS, ULS, or minres; e.g, Harman & Jones, 1966), alpha factor analysis (Kaiser & Caffrey, 1965), Jöreskog's (1966; 2007) image factor analysis; and multiple independent cluster analysis (MICA; following Guttman, 1952).
- *Component analysis:* Principal components (Hotelling, 1933), Harris (1965) component analysis, and MICA are available.
- *Image analysis:* Guttman's (1953) image analysis is available, as well as MICA.

Confirmatory Features

- *Multiple independent cluster analysis (MICA).* With most methods the user may pre-specify factor or component weights by including a variable weight matrix. MICA options are especially useful for cases in which independent clusters of variables or items are hypothesized—where each variable loads on one and only one factor. The method was described by Guttman (1944; 1952), and independently formulated by Holzinger (1944) as "multiple group" factor analysis[1]. MICA may be useful in test construction applications in which items are expected to form homogeneous but correlated clusters.
- *Target rotation.* Hypercon (Fleming, 2012) uses a different rationale from MICA. Whereas the latter was designed for cluster-oriented solutions, hypercon is a more general procedure based on the early work of Horst (1941) in hyperplane fitting, and extended by Derflinger and Kaiser (1989) and Fleming (2012). While MICA is "extraction" based, hypercon is "rotation" based. Promax target rotation (Hendrickson & White, 1964) uses a pre-specified target matrix.

[1] Also known as oblique multiple group factor analysis (OMG; Stuive and others, 2008). I prefer the name multiple independent cluster analysis simply because most people think of "multiple groups" as referring to different groups of people rather than to clusters of variables.

MICA requires entering a variable weighting matrix whereas hypercon and promax require entering a target matrix for rotation (see Figure 2).

Input and Dispersion Matrices

The program accepts raw data from which a correlation, covariance, or average cross-products matrix is formed, or a correlation or covariance matrix may be input directly. Either of the latter may be factored but by tradition, exploratory factor and component analysis favor the use of correlations. Harris component analysis, image analysis, and image factor analysis factor utilize special covariance matrices that are computed automatically when these options are selected. Ordinary least-squares requires that a correlation matrix be factored. Table 2 shows permissible combinations of models, methods and matrices.

A loading (factor pattern) matrix to be rotated by any one of several methods may also be input rather than a raw data or correlation/covariance matrix.

Rotations

Explorer provides a number of factor rotations, including all of the orthomax class of quartic orthogonal rotations (such as varimax, quartimax, equamax, and parsimax; see Crawford & Ferguson, 1970; Clarkson & Jennrich, 1988). For oblique rotations Harris and Kaiser‘s (1964) orthoblique, promax (Hendrickson & White, 1964), oblique quartic rotations such as oblimin (Jennrich & Sampson, 1966) and the Crawford & Ferguson family of (1970) rotations, and several hyperplane fitting methods are available (Fleming, 2012). Bi-factor rotations and others using gradient projection algorithms (Bernaards & Jennrich, 2005) are included as well.

Additional Features

- *Number of factors.* The number of factors (or components) can be determined by several methods.
- *Communalities.* These can be estimated by several methods for the common factor model.
- *Univariate and multivariate outlier detection.* Available when raw data are input.
- *Statistics and fit measures.* A number of univariate and multivariate statistics may be requested when raw data are input for both descriptive purposes and to help in identifying outliers. Also, several goodness of fit measures are available for interpreting and evaluating results.
- *Residuals analysis.* Extensive residuals analysis can help to spot variables, or variable pairs, for which factoring is incomplete.
- *Factor scores.* Factor score estimates by all leading methods may be requested.
- *Graphic displays.* Graphic plots of eigenvalues, factor loadings, and residuals are available.
- *Second-order and hierarchical factors.* Second-order factors and the Schmid-Leiman (1957) solution for hierarchical factors are available.
- *Bifactor analysis.* Available as of release 3.3, per Jennrich and Bentler (2011; 2012).
- *Missing data.* Explorer includes missing data options that are useful when a few cases or values are randomly missing.

All of these features are described in greater detail further on in this guide.

Comparisons with Commercial Software Packages
Commercial software packages such as SAS™ , STATA™, and IBM SPSS™ perform many of the same factor and rotation procedures as Explorer, yet each offers unique features as well. Explorer does (for example) more rotations than those found in those programs (see Tables 5a, 5b, and 8). Explorer also offers some factor scale and item analysis features not found in other factor analysis programs; and Explorer is still under development, so more features are planned for future releases.

The Explorer Command File and Explorer Command Language (ECL)
A command file is used to specify the flow of processing. This is a free-format file (entries are not required to be in specific columns or fields) but maximum record length is 80 characters. The command file is organized into paragraphs into which specific command statements are entered (e.g., cases = *number*; in which the number of cases is specified). Taken together the syntactical structure forms a kind of language: Explorer Command Language, or ECL. But the syntax is simple and easy to learn—especially from the examples given here.

Executing Explorer

Explorer runs interactively in "command" mode where the user is asked, simply, for the number of variables and names of the command and output (listing) file names. The user is first greeted by the message:

Running EXPLORER . . .

Next, the user is prompted for the number of variables and the necessary file names with:

Number of variables:	[Total number of variables read]
Command file:	[Name of file containing ECL processing commands]
Listing file:	[Explorer output or "results" file]

If there are any errors, the user is so notified on the screen, as in:

***** Program halted due to errors.**

Additional error information may appear on the screen, but in general more detailed error messages and warnings are printed on the listing file.

A normal end of run is signaled by the message:

. . . All Done!

The Explorer Command File and Command Language

1. Basics

An Example

Figure 1 is provided because a pictured example is truly worth 1,000 words. It illustrates a typical command file for running Explorer. Note that comments are entered in curly brackets {}. These are for documentary purposes only and are not executed.

Command File Structure: Paragraphs and Statements

Explorer commands are organized into paragraphs. The paragraphs and the command statements within them of the command file are order independent, except for the final <end> paragraph—which contains no statements—but logical considerations suggest an ordering like the example in Figure. 1. Comments, however, may be placed wherever they are desired.

```
<headers>                          {Headers (1 to 3 lines) are optional}
  'Box Problem from Thurstone (1947) for 20 Variables';
  'Uses Data Matrix from Kaiser and Horst (1975)';
  'OLS Factoring with Oblique Transformation';

<data>
   cases = 40; matrix = raw;  file = 'c:\mydata\box20.dat';
{There are 40 cases; a raw data matrix is input; the format is
     'freeform' by default; and the path to the data file is given.}

<analysis>
     method=ols;                   {Ordinary least-squares is the factoring method.}
     matrix = corr;                {The correlation matrix is analyzed.}
     trans = cfvm;                 {The transformation is Crawford-Ferguson oblique
                                   varimax.}
     com=smc;                      {Squared-multiple correlations are initial communalities.}
     number =3;                    {Number of factors is pre-specified to be 3.}

<plots>
    eigen;                         {Sree plot of eigenvalues (default).}
    trans = 3;                     {Plot first 3 obliquely transformed factor pairs.}

<output>
    all;                           {Maximum output is to be printed.}

<end>                             {This is always the last paragraph—contains no statements
                                   but it is required.}
```

Figure 1: Example of an Explorer Command File and

Explorer Command Language (ECL)

The maximum record length for the command file is 80 characters (spaces); any statement that is longer than this length can simply be continued on the next line. Within this limitation statements and paragraph names may appear freely in any position.

Embedded blanks are ignored (except when placed within single quotation marks, like this: ‘variable 1’).

Paragraph names are *placed within angle brackets*. Example:

<data>	(this is the beginning of the data paragraph)

Statements within paragraphs are the “sentences” that define the analysis. *Statements always end with a semicolon*. Example:

file = ‘c:\mydata\myfile.dat’;	(specifies the name and location of the data file)

Paragraph names and statements are not case sensitive. Examples:

<data>	is the same as:	<DATA>	(name of data paragraph)
CASES = 100;	is the same as:	cases = 100;	(statement tells that there are 100 cases)

Comments

Comments may appear anywhere in the command file. Comments can be useful for documenting procedures for future reference. Comments are enclosed in *curly brackets*, as in: {This is a comment.}

Headers Paragraph

Explorer allows up to three lines of heading that are printed at the top of each page of output for documentation purposes. These are enclosed in single quotes in the command file. The maximum length of a header is 65 spaces. Examples:

```
<headers>
   ‘Analysis of Smith Learning Data’;
   ‘Principal Factor Analysis with Orthoblique Rotation’;
    ‘Collected May, 2009’;
```

Data Paragraph

This paragraph specifies basic processing parameters, such as the number of cases, the form of the input data matrix (raw, correlation, or covariance), Fortran type variable format for reading the data if needed, and the name of the file containing the data. Possible entries are shown in the Table 1.

Table 1
Statements for the Data Paragraph

Statement	***Interpretation***
cases = *n;*	The number of cases (*n*) may be given, even if input matrix is a correlation or covariance matrix, because this value may be used in certain statistical computations. Conversely, the number of cases can be computed automatically when data are read from the raw data file—in which case the statement is not needed.
file= '*filespec';*	The name of the input file containing the raw data, covariance, or correlation matrix—including the path to the file, if not in the current directory—should be enclosed in single quotes.
format= '*(format)';*	A Fortran-type variable format for reading the data file should be specified unless the data are in "free" format—that is— the data values are separated by blanks. In the latter case, use: format = *; (or specify nothing, as this is the default). Format specifications are embedded in single quotes and begin and end with parentheses, as in: '(5x,20f3.0)';
matrix = *matrix;*	This refers to the matrix that is read into the program for analysis. The options are raw, correlation, covariance, and loading.
boots = *nboots;*	For bootstrapped standard errors (refer to Appendix A for details).
shape = *shape;*	The shape of the correlation/covariance matrix that is input can be full (fu), lower triangular (lt), or sub-diagonal (su). The latter only applies to correlation matrices. Full is the default.
missing = *value;*	Missing data are indicated by blanks (hence data must be read by a format when there are missing values). There are two options for *value*: case deletion (cd) and mean replacement (mr). (These options are useful when there is a small number of missing data points that are missing randomly.)
wtfile*=*filespec*;	Specifies location of the weight file for the MICA options.
tfile*=*filespec*;	Specifies location of the target file for hypercon or promax rotations.
save;	Causes matrix output (matrix to be factored and loading matrices, and factor intercorrelations if oblique) to be output to a file named MATRICES.LIS.

*****These files are always in "freeform" format. See Figure 2 for examples.

Table 2
Permissible Combinations of Method and Matrix for Each Model (Analysis paragraph)

Model	***Specs for* method=**		***Specs for* matrix=**
Common factor analysis	pfa;	Iterated principal factor analysis* (principal axes)	Correlation** or covariance, (cor or cov)
	ols; uls ; or minres;	Least squares	Correlation (cor)
	jk;	Jöreskog's image factor analysis	Correlation (cor)±
	ml;	Maximum likelihood	Correlation or covariance (cor or cov)
	alpha	Alpha factor analysis	Correlation (cor)
	mica1;	MICA	Correlation (cor)
Component analysis	pca;	Principal components	Correlation**, covariance, or cross-products† (cor, cov, or xp†)
	harris;	Harris components	Correlation (cor)±
	mica2;	MICA	Correlation (cor)
Image analysis	image;	Image analysis	(Partial) image covariance matrix (cor)±
	mica3;	MICA	(Same as above)

*Default method. **Default matrix for this method.

±These methods actually factor a special kind of covariance matrix, but computations begin with correlations, and the methods are scale-free so that beginning with correlations is a convenience rather than a necessity.

†Matrix of average sums of squares and cross-products (ASSCP) of raw scores.

Help Box 1:
Which Combination of Model and Method Should I Choose?

This is not a easy question to answer. It depends on many factors, including the researcher's knowledge of factor analysis methods, and whether the research is more exploratory or confirmatory (within the context of exploratory factor analysis). If the research is designed to test very specific hypotheses regarding the factor structure then a structural equation modeling (SEM) program should be considered rather than Explorer.

Image analysis is a theoretically interesting method, but mainly to students of factor analysis rather than to the average researcher. The same can be said of Harris component analysis. On the other hand, alpha factor analysis and Jöreskog's image factor analysis are useful methods of common factor analysis that have been underexploited.

For most researchers, the issue comes down to a choice between component analysis and factor analysis. As it happens, principal components and principal factors often produce results for which the factor patterns look similar, though there are cases where the differences are notable. But the statistical theory underlying these two models is quite different. A good discussion of the differences can be found in Widaman (2007).

The author's preference, by background and training, is for the common factor model. When the scale of measurement is arbitrary, as with many psychological variables, correlation matrices are more convenient than covariance matrices; and today's methods make analysis of correlation structures (including exploratory factor analysis) statistically respectable (Bentler, 2007).

Principal axis factor analysis is the most commonly used method. Aside from this, alpha, and Jöreskog's method, there are several others for the common factor model. These include unweighted least-squares (or ULS, also called ordinary least squares, or OLS), and maximum likelihood (ML). Although maximum likelihood has been the favored method among statisticians, ML can be problematic. In a study of estimation methods, MacCallum, Browne, and Cai (2007) found that "… it rarely occurred that ML recovered [weakly defined] factors better than did OLS, and it never occurred that ML recovered [a weak factor] accurately while OLS did not" (p. 165). On the other hand, the statistical theory for ML is more highly developed than for other methods (e.g., in facilitating the computation of standard errors).

The foregoing are a few of the author's thoughts; but each person must decide which combination of methods is most suitable for his/her needs.

Analysis Paragraph
This paragraph drives the analysis. The method, matrix to be factored (as opposed to the input matrix), the rotation and transformation are specified, as are the communality estimation method and the method for determining the number of factors. (A transformation refers to an oblique transformation of axes, as opposed to an orthogonal rotation, because in some cases oblique analysis follows a preliminary orthogonal rotation.) There are several tables following that specify the syntax used in the data and analysis paragraphs.

Method of Factoring and Matrix to be Analyzed
These are summarized in Table 2. (For example: method=pfa; matrix=cov;).

Rotations: Orthogonal rotations are summarized in Table 5a; oblique transformations in Table 5b. Note that some of these are really combinations of others. For example, varimax, quartimax, equamax, and parsimax are all special cases of the more general orthomax rotations in which the gamma parameter is defaulted; so these can be considered convenient shortcuts. The same may be said of some of the oblique rotations, where direct quartimin is a special case of direct oblimin in which the delta parameter is preset; and so forth with weighted oblimin, etc.

To obtain an orthogonal rotation, the syntax is "rotate=*rotation*;" where the latter parameter is the type of rotation. Example:

rotate=vmax;

for varimax. For oblique transformation the syntax is "trans=*rotation*;" as, for example,

trans=dq;

for quartimin (aka direct quartimin).

Random starts : By default, Explorer produces 50 random orthogonal starts for rotations. To change this, specify:

Random = *number*; (for example, random=0 to suppress random starts).

Kaiser normalization: To suppress Kaiser normalization prior to rotations (default for all except weighted varimax), use the statement:

nonorm;

Communalities and number of factors options are given in the following tables. Table 3 describes communality options and the number of factors options are shown in Table 4—but see the box following regarding the determination of the number of factors.

Second-order factors
Use "secord;" to obtain a second-order factor analysis, including Schmid and Leiman's (1957) hierarchical loadings. Optionally, use "nsec=number;" to specify the number of second-order factors. (Second order factors are currently computed only using pfa, ols, and pca factoring.)

Table 3
Communality Options (Analysis Paragraph)

Value	***Explanation***
com = smc;*	Squared-multiple correlations.
com = bigr;	Largest absolute off-diagonal element for each row.
com = lso;	Largest squared off-diagonal element.
com = unity;	Ones are placed in the diagonal of the correlation matrix.
diag = *value1, value2, …*;	A list of numeric values is placed in the diagonal. These are separated by commas.

Note: Communalities apply only to the common factor model. Initial communalities when a covariance matrix is factored are variances (SMCs and unity do not apply).
*This is the default for communalities when a correlation matrix is factored.

Table 4
Number of Factors Options (Analysis Paragraph)

Value	***Explanation***
number = *p*;	The number of factors ($p > 0$) is pre-specified. Must be $\leq$ the number of variables; strictly < this number for common factors.
number = kg;	The number is determined by the Kaiser-Guttman rule (eigenvalues of correlation matrrix > 1.0).
number = 0; (default for number of factors)	The Zoski & Jurs (1996) method of approximating Cattell's (1966) scree test is used to estimate the number. (If the number of variables is < 6, however, the program defaults to kg.)
ecrit = *value*;	The number of factors retained is equal to the number of eigenvalues > value (e.g., ecrit = 2.0).
pct = *percent*; (technically applies only to PCA)	The value here is a percentage between 1.0 and 100.0 in which the number of *components* retained "accounts for" this percentage of variance. (E.g., pct = 75;). (FA model does not "account for variance" – it "accounts for covariance.")

Labels Paragraph
Variables may be given labels or short names of up to eight characters. Labels are listed in the order in which the variables are input, though variables can later be re-ordered if desired. Labels are enclosed in single quotes, so that they are case sensitive and may include blanks or special characters. Example (for five variables):

<labels>
‘age’; ‘test 1’; ‘test 2’; ‘test 3’; ‘final’;

Help Box 2:
An Important Note Regarding the Number of Factors

Options for the number of factors are shown in Table 4. Deciding on the correct number of factors is probably the most important decision to be made in using factor analysis and the related models—more crucial, in fact, then the method of extraction or of rotation—and perhaps even more important than the choice of model. Yet many people rely on computer software to determine this number, or rules of thumb such as the Kaiser-Guttman rule (factors or components are retained when corresponding eigenvalues of the unaltered correlation matrix are > 1). If researchers haven’t some more rational reason then it is a good idea to explore the data by extracting different numbers of factors, then deciding based on interpretability, or perhaps by a careful scrutiny of the scree (eigenvalue) plot, which is output by Explorer by default.

With this caveat in mind (I carefully considered whether there should even be a default), Explorer does have a default option, the Zoski & Jurs (1966) method, which is, as its title suggests, “An objective counterpart to the visual scree test …” This method seems to work well in many circumstances, and in any case, can provide a good starting point for further exploration of the data.

Select Paragraph
This paragraph is used to select variables and cases. Note that values in lists are separated by commas. Examples are:

case_deletion = 23, 24;	Deletes cases with ordinal numbers in the file of 23 and 24.
reorder = 1, 3, 5, 9, 7, 10;	Assuming 10 variables this omits numbers 2, 4, 6, and 8, and also switches 9 and 7.
delete = 3, 6, 9;	Use all variables on the list except 3, 6, and 9.
keep = 1, 6, 8, 10, 12, 15;	Keep is the inverse of delete because it specifies which variables to include rather than which to exclude.

Note: Only *one* of the latter three select options—reorder, delete, or keep, may be specified for a given analysis as these are mutually exclusive. Reorder is the most general because it tells the program which variables to use, which to drop, and in what order they should appear.

Table 5a
Summary of Orthogonal Rotation Options (Analysis Paragraph)

Class[*]	*Rotation type*	*Code*	*Modifying parameter*	*Parameter Default*	*Source*
A	Varimax[**]	vmax;			Kaiser (1958)
A	Weighted varimax	wvmax;			Cureton & Mulaik (1975)
A	Quartimax	qmax;			**
A	Biquartimax	bi;			***
A	Equamax	emax;			Saunders (1962)
A	Parsimax	pars;			Crawford & Ferguson (1970)
A	Factor parsimony	fp;			(As above)
A	Orthomax	omax;	gamma = (value);	1.0[†]	(As above)
B	Bi-factor quartimin	bqmn;			Jennrich & Bentler (2011)
B	Bi-factor minimum entropy	bmnt;			(As above)
C	Minimum entropy	ment;			Jennrich (2004)
C	Orthosim	osim;			Bentler (1977)

[*]*Classes: A = orthomax, B = Bi-factor, C = Other.*
[**]Varimax is the default orthogonal rotation.
[**]Four papers that appeared at about the same time introduced quartimax, or rotations that were essentially equivalent (Carroll, 1953; Ferguson, 1954; Sauders, 1953; Neuhaus & Wrigley, 1954).
***Orthogonal equivalent of biquartimin (see Table 5b).
[†]Equivalent of varimax.

Table 5b
Summary of Oblique Rotation (Transformation) Options (Analysis Paragraph)

Class*	***Type***	***Code***	***Modifying parameter***	***Parameter Default***	***Source***
A	Direct quartimin	dq;			Jennrich & Sampson (1966)
A	Direct oblimin	dobl;	delta=*value;* (value ≤ 0.)	0.	Jennrich & Sampson (1966)
A	Weighted oblimin	wobl;			Lorenzo-Seva (2000)
A	CF[†] "oblique varimax"	cfvm;			Crawford & Ferguson (1970); Clarkson & Jennrich (1988)
A	CF[†] "oblique quartimax" (aka quartimin)	cfqu;			(As above)
A	CF[†] "oblique equamax"	cfeq;			(A above)
A	CF[†] "oblique parsimax"	cfpa;			(As above)
A	Biquartimin	cfbi;			(As above)
A	Covarimin	cfco;			(As above)
A	Factor parsimony	cffp;			(As above)
B	Orthoblique "Class II"	orthob;	power = *value*;	0.5	Harris & Kaiser (1966)
B	Orthoblique: independent clusters	inclus;			(As above)
C	Promax	pmax;	power = *value;*	4	Hendrickson & White (1964)
C	Weighted promax	wpro;	power = *value*;	4	Cureton & Mulaik (1975)
C	Promax target (confirmatory)	target;			

C	Promaj	pmaj;			Trendafilov (1994)
C	Casey (or KC)	kc;	power = *value;*	4	Kaiser & Cerny (1978)
C	Horst-Hilsch	hh;	power = *value;*	4	Derflinger & Kaiser (1989)
C	Hypermax	hyper;	power = *value*;	4	Fleming (2012)
C	Hyperalt**	ha;	power = *value*;	4	(See footnote)
C	Hypercon target (confirmatory)	hcon;			Fleming (2012)
D	Oblisim	osim;			Bentler (1977)
D	Geomin	geo;	geps = *value*;	.01	Yates (1987), Browne (2001)
E	Bi-factor quartimin	bqmn;			Jennrich & Bentler (2012)
E	Bi-factor geomin	bgeo;			(As above)

*Classes: A = Quartic/analytical (Clarkson & Jennrich, 1988); B = based on rescaled eigenvectors (Harris & Kaiser, 1966); C = procrustes/hyperplane fitting; D = other; E = Bi-factor.

**Hyperalt uses direct oblimin as the preliminary rotation. At present it is experimental.

†CF = Crawford/Ferguson.

Table 6
Factor Scoring Options* (Analysis Paragraph)

Value	***Explanation***
fscore;	For component and image analysis, requests that scores be computed directly. For factor analysis estimated scores have several choices but the default is fscore=regression;
fscore=regression;	Common factor analysis only: Scores are estimated by the regression method. (Same as just fscore; with no argument.)
fscore=bartlett;	Common factor analysis only: Scores are estimated by Bartlett's method.
fscore=ideal;	Common factor analysis only: Scores are estimated by the ideal variables method.

Note: Factor scores are output to a file called 'FSCORES.DAT'.
*A good discussion of these can be found in Harman (1976).

Output Paragraph

The output paragraph allows users to select or limit the amount of printed output. For minimal output (communalities and final loadings) use the keyword:

min;

For all (maximal) output, use:

all;

Refer to Table 7 for more exact details on output options. Typical examples might be:

<output> min; cor; univ;	(Minimal output as shown in Table 7, plus correlation matrix and univariate statistics.)
<output> pcor;	(Default output, per Table 7, plus partial and multiple correlations.)

(*Note:* Explorer (i.e., your author) really wants to give you the correlation matrix with scree plot and associated eigenvalues, even if you are analyzing a covariance matrix and have already decided on the number of factors. These can, of course, be suppressed.)

Plots Paragraph

This paragraph specifies optional plots that may be requested. Options are:

<plots> eigen;	(Scree plot of eigenvalues. This is the default, unless minimal output is requested—see Table 7.)
init = 3;	(Plots first three pairs of factor loadings for initial solution.)
rotate = 3;	(Plots first three pairs of factor loadings for orthogonal rotation.)
trans = 4;	(Plots first four pairs of factor loadings for oblique solution.)

Residuals plots do not need to be requested as they are produced automatically when residuals analysis is requested in the output paragraph.

Details Paragraph

Most users will seldom need the features provided in the <details> paragraph but there are a few parameters that occasionally my need changing. These include altering the maximum number of iterations for extracting factors and for rotation, the convergence criteria for these processes, and the number of decimal points printed for matrix output. Also, the values for identifying salient versus hyperplane elements to be used when sorting loadings can be specified. The defaults are 500 iterations for extraction and 500 for rotation, with epsilon values of .000001 for convergence. The default number of decimal places is 3. The default for salient loadings is 0.395 (rounds up to .40) and 0.204 for hyperplane elements. These can be changed using the following statements:

<details>

maxit = *value*;	(Set maximum iterations for extraction—default = 500.)
maxrot = *value*;	(Set maximum iterations for rotations—default = 500.)
fconv = *value*;	(Set convergence criteria for extraction—default = .000001.)
rconv = *value*;	(Set convergence criteria for rotation—default = .00001)
ndec = *value*;	(Set no. of decimal places [2- 4] for matrix output—default = 3.)
hival = *value*;	(Value considered “large” for loadings—default = .395.)
loval = *value*;	(Value considered “zero” for loadings—default = .204.)

1 0 0	**1 0 1**	**1 0 0**
1 0 0	**1 0 0**	**.8 0 0**
1 0 0	**1 0 0**	**.8 0 0**
1 0 0	**1 0 0**	**.7 0 .3**
0 1 0	**0 1 1**	**0 1 0**
0 1 0	**0 1 0**	**0 1 1**
0 1 0	**0 1 0**	**0 .5 -1**
0 1 0	**0 1 0**	**0 .9 .2**
0 0 1	**1 0 1**	**0 0 1**
0 0 1	**0 0 1**	**.4 0 .8**
0 0 1	**0 0 1**	**0 0 -.6**
0 0 1	**0 1 1**	**0 .3 .9**
(a) MICA weight matrix	**(b) Hypercon target matrix**	**(c) Promax target matrix**

Figure 2: Examples of Explorer MICA Weight Matrix, and Hypercon and Promax Target Matrices

In all examples there are 3 factors and 12 variables. In the MICA weight matrix (a), weights of 1 means that those correlations/covariances are simply summed for the corresponding factor. (It is also assumed, for convenience, that variables are ordered so that the first four load on the first factor, and so forth.) Although these weights can take on any values it makes most sense for them to be constants of equal value if multiple independent clusters are hypothesized.

For hypercon (b) only binary values that should be used: ones (for salient loadings) and zeros (for loadings in the hyperplane).

With promax the target matrix can take any form, corresponding roughly to the predicted magnitude of the loadings. A promax target could also look like either (a) or (b).

In all cases the weight or target matrix must be of full column rank.

End Paragraph
The last paragraph consists of just the paragraph name, which signals the end of the input stream for the command file. The end paragraph is mandatory. It looks like this:

<end>

2. Technicalities[2]

Abbreviations
Paragraph names and keyword within statements may be abbreviated. Two characters is the minimum. Two characters will work provided these uniquely define a keyword within its respective paragraph. (Note an exception—for the "matrix =" statement in the analysis paragraph at least three characters are needed to distinguish correlation from covariance: cor or cov.)

Examples:
<AN>, <ANAL>, etc., may be used to abbreviate <ANALYSIS>.
ma = , mat =, etc., may be used to abbreviate matrix =.

Note that spelling errors are often forgiven, so that, for example, writing <anlasis> for <analysis> still works, because the string is "about" the right length and the first two letters are correct.

Free-Form Command File
Paragraph names and statements within paragraphs may appear in any position on the command file within the 80-column input field. It is only for neatness and readability that paragraph names begin at the left margin and statements appear indented, beginning on the next line, in many of these examples.

Lower-Upper Case Notation
The syntax is not case-sensitive; in other words, either upper or lower case paragraph names or statements may be used—except when values (such as variable names or headers) are enclosed within single quotes.

Spaces (Blanks)
Spaces or blanks are ignored in parsing the input file, unless they are enclosed in single quotes. An example of the latter is a variable name with an embedded space: 'Time 1'.[3]

---

[2] This section is technical and is therefore optional.

[3] Internally, the ECL parsing routine (a) eliminates blanks, and (b) converts all text to upper case—the exceptions in both cases being text enclosed in single quotes, and comments.

Format of Statements
Statements can take these forms:

keyword;
keyword = value;
keyword = value1, value2, …;
value1[, value2, …];

A keyword standing by itself is used to request something.

<output> min; evec;	(min; requests minimal output, plus eigenvectors (evec;))

Some keywords are followed by a value in order to set basic parameters for processing. This was seen for example above in the <data> paragraph, as when the number of cases was specified:

cases = 225;

An example of a keyword followed by a list of values is:

diag = .60, .80, .78, .59, .35;

Here the user is entering starting values for communalities in the <analysis> paragraph.

A simple list of values *not* preceded by a keyword was seen in the labels and header paragraph; refer back to those for an illustration.

Default Values
Statements omitted from a paragraph will be defaulted. For example, if no labels are included, variables are labeled Var 1, Var 2, etc. And some paragraphs (e.g., labels or headers) may be omitted entirely when defaults are taken.

No Null Paragraphs (Except for <END>)
Some paragraphs may be omitted (e.g., <HEADERS>; <LABELS>), but if paragraphs are present, they must have at least one statement ending with a semicolon—the exception being the <END> paragraph, which has no statements.

Order-independent Variable Labels
In the <labels> paragraph some but not all variables may be labeled by using the order-independent method, in which variable names are preceded by their ordinal index and an equal sign, as in this example:

2 = ‘height’; 5 = ‘girth’; 3= ‘weight’;

Table 7
Output Paragraph Options

Keyword	*Resulting Output*	*Minimal (min;[**])*	*Default*	*All (all; [***])*
univ;	Univariate statistics with outlier identification	N	Y[*]	Y[*]
mult;	Multivariate statistics with outlier identification	N	Y[*]	Y[*]
cor; or cov;	Correlation or covariance matrix (as applicable)	N	Y	Y
pcor;	Partial and multiple correlations	N	N	Y
rinv;	Inverse of correlation matrix	N	N	N
anti;	Anti-image correlation matrix	N	N	N
msa;	Measures of sampling adequacy	N	N	Y
eval;	Eigenvalues	(Always output)	(Always output)	(Always output)
evec;	Eigenvectors	N	N	N
[none]	Initial and final communalities	(Always output)	(Always output)	(Always output)
init;	Initial factor solution (loadings) and percentages of variance	N	Y	Y
[none]	Rotated loadings (pattern if orthogonal; also structure & factor correlations, if oblique)	(Always output)	(Always output)	(Always output)
refr;	Reference structure (oblique).	N	Y	Y
rfcor;	Reference factor correlations (oblique only)	N	N	N
resid;	Residuals analysis	N	Y	Y
rot;	Factor rotation/transformation matrix (not loadings)	N	Y	Y
sort;	Sorted loadings (elements of pattern matrix) and factor scale statistics (Lorenzo-Seva, coefficient omega, etc.)	N	Y	Y
ifs;	Indexes of factor simplicity	N	N	Y
item;	Factor scale and item analysis	N	N	Y[*]
wt;	Factor weight matrix	N	Y	Y

[*]Applies only when raw data are analyzed. [**]This is an order-dependent parameter that should come first in the output paragraph when used. [***]Includes everything except inverse of correlation matrix, eigenvectors, anti-image correlation matrix, and reference factor correlations.

Help Box 3:
A Word or Two about Interpreting the Matrices of Factor Analysis

As is well-known, for orthogonal factor solutions the factor pattern matrix—containing the weights for regressing the factors on the variables—is identical with the factor structure matrix—which contains the correlations between the variables and factors. But in oblique solutions these two matrices are not identical. Oblique solutions also produce a so-called reference structure matrix, which is a matrix of part correlations between the factors and each variable with the effects of the other variables removed. All three of these matrices—pattern, structure, and reference structure—are sometimes referred to as "loading matrices," so that this term is notoriously ambiguous when the factors are oblique (Harman, 1976). Indeed, some writers have advocated the elimination of the term "factor loadings" altogether because of this confusion. However, while it is always essential to clarify exactly which matrix one is presenting in a research report, the general term "loading" is likely to remain with us—it is too deeply ingrained in the psychological literature and still used by psychometricians today. By factor loadings, psychometricians ordinarily mean elements of the factor pattern, although historically Thurstone often used the term to describe elements of the reference structure.

So, in oblique analysis, which of these matrices is the most useful for interpreting the factors? This is a matter of some controversy. Some favor the factor structure because (1) these are correlation coefficients and are therefore easier to interpret than regression weights, and (2) the factor structure matrix has analogues with other multivariate methods. It is also true that, when the underlying structure is poorly understood and a simple structure solution does not exist, then this structure matrix can be invaluable to factor interpretation.

On the other hand, consider that the factor structure matrix is equal to the product of the pattern matrix times the matrix of factor intercorrelations, or: $\mathbf{S} = \mathbf{PR_{ff}}$. When there truly is a very simple structure of correlated clusters of variables, then structure matrix **S** conflates the interpretation because the near zero elements of the factor pattern will generally be lower than the corresponding elements of the structure matrix; or in other words, the structure matrix confounds the simplicity of the solution with the closeness or "correlatedness" of the factors. Better to interpret the pattern and structure separately in such cases, in my opinion.

If you would like to "have your cake and eat it too," some analysts (Thurstone; Kaiser) have advocated interpreting the reference structure (rather than the pattern) because (1) the elements are correlations rather than regression coefficients, and (2) the reference structure matrix is column-wise proportional to the factor pattern. Therefore, in a relative sense, the same high, middle, and low loadings will be found in both. If one considers the scaling of the columns to be arbitrary, then this matrix can be quite interesting.

The factor intercorrelation matrix, produced in oblique analysis, is always of interest. But the intercorrelation matrix of reference factors is useful only in plotting reference factors alongside primary factors for heuristic purposes—and is never interpreted otherwise.

Then there is the factor weight matrix, which contains the coefficients used to estimate the factor scores. This matrix is difficult to interpret as the values can fluctuate due to multicollinearities in the variables. Even when the pattern matrix exhibits a very simple structure solution the factor weights may not reveal such a structure. The weight matrix will also vary considerably according to the method used to estimate factor scores. (This is not a problem with component and image analysis, however, as the scores for those methods are fixed and determinate).

Statistical and Psychometric Measures Available with Explorer

Univariate and Multivariate Statistics

These can be requested when raw data are input. Univariate statistics include for each variable: mean and standard deviation, variance, standard error of the mean, coefficient of variation, number of valid cases, and skewness and kurtosis measures.

Multivariate statistics include skewness and kurtosis and approximate z-values (Mardia, 1972), mean Mahalanobis distance, Bartlett's (1950) test for complete independence of the correlation matrix (aka, sphericity test), and the determinant and generalized variance of the correlation matrix.

Outlier Identification (for Raw Data Only)

When univariate statistics are requested, cases that are potential outliers are identified by the most extreme standard scores. Cases with largest Mahalanobis distance measures are identified in the multivariate case. When cases are judged to be outliers they may be eliminated from subsequent runs by using the case deletion feature in the <select> paragraph.

Correlational Measures

Correlation coefficients may be printed as well as the inverse of the correlation matrix, a matrix of partial correlations with multiple correlations on the diagonal, and Kaiser's (1981) measures of sampling adequacy for each variable. The determinant and generalized variance are also available (see multivariate statistics). The anti-image correlation matrix may also be requested.

Model Fit

Approximate chi-square measures of factor fit (as sample size increases these become more accurate) and degrees of freedom are given for maximum likelihood and image factor analysis (Jöreskog, 1962; 2007). (Also see Residuals Analysis.)

Communalities and Uniquenesses

Initial and final communalites, and unique variances, are routinely output along with their means and medians.

Residuals Analysis

When residuals analysis is requested the residual correlation/standardized covariance matrix is printed along with a table of largest residual pairs, and a residuals plot.

"**Goodness" of Factor Solution Based on Loadings**

Sorted loadings may be requested in which the most salient are shown with smallest values set to zero.

Indexes of factor simplicity for each variable (Kaiser, 1976), and their means and medians may be requested, along with values in hyperplane bands of various sizes (e.g., [-.10, +.10] and the Kaiser and Cerny (1978) hyperplane fit measures.

Measures of simple matrix structure due to Bentler (1977) and Lorenzo-Seva (2003) are included with factor loadings.

Factor Scale Measures

Coefficients alpha (raw and standardized) may be requested for each factor based on the items with the highest loadings on the factor, as well an item analysis for these items (item-total correlation and alpha if item is removed). As of v. 3.2, confidence intervals for alpha are also output. Alpha is only available with raw input data. McDonald's coefficient omega is also available as of release 2.0.

Fleming's (2003) scale fit indexes are also included when factor scale measures are requested.

Help Box 4:
A Few Thoughts on Rotations

Rotations of initial factor solutions are designed to improve interpretability by approaching Thurstone's (1947) ideal of simple structure. His five principles (see, e.g., Harman, 1976, p. 99) describe this concept, but to oversimplify a bit, a good solution should have many loadings that are zeros—or in practice—differ from zero by only trivial amounts. Most measured attributes in nature, whether human intellectual abilities or the sizes and shapes of primate bones, are correlated; thus by implication correlated factors (oblique rotations) are likely to give simpler structure than orthogonal rotations. For this reason oblique solutions are favored by most psychometricians (if not always by researchers).

(Incidentally the term *rotation* in mathematics refers to a rigid shifting of axes to maintain a 90° separation, and by tradition applies only to the orthogonal case. Technically the term *transformation* of axes is appropriate for the oblique case in which this angular criterion is relaxed. But by common usage, the term rotation has come to be applied to both the orthogonal and oblique cases.)

It might be said that there are "good enough" rotations and then there are "better" rotations. If the goal of the factor analysis is validation of tests or constructs then many users are satisfied if there is an arbitrary separation of high loadings (e.g., with cutoff values of .30 or .40) from low ones. For this purpose, many papers are still being published based on the standby orthogonal varimax rotation. If the variables are of low complexity (load mainly on one factor) by design, as is often the case in test development, then varimax or probably any of the rotations available in Explorer will satisfy the researcher and journal editor. Using an orthogonal rotation, of course, ignores the fact that the underlying concepts are almost certainly correlated, so many factor analysts (the author included) prefer an oblique solution, if only for this reason.

But the main reason to prefer oblique rotations is that they can more closely approximate simple structure. In psychometrics the trend is to prefer oblique rotations that clearly locate hyperplanes. A geometric (and rather technical) explanation of this is given by Harman (1976, p. 87), but to once again oversimplify, when hyperplanes are properly fit there will be (a) many essentially zero loadings, and ideally, (b) a clear separation of these from the non-zero loadings. Moreover, hyperplane fitting rotations attempt to maximize the hyperplane count, or number of zero loadings (Fleming, 2012).

A related reason for preferring hyperplane fitting rotations is that they are better at uncovering simple structure in complex data (when variables load on more than one factor) as in Thurstone's (1947) 26-variable box problem (Fleming, 2012). In this problem and in many other examples, the hypermax rotation was shown to perform at least as well as any other, and better than most; and hypermax is also a very efficient method in terms of computer speed (Fleming, 2012).

However, no rotation is perfect for every occasion (Browne, 2001; Fleming, 2012), so it is often a good idea to try several, but especially when the project is very much an exploratory one with complexities in the variables and an unknown factor structure.

A Note on Algorithms[4]

For extracting latent roots and vectors of dispersion matrices, Explorer employs a QR algorithm of a Householder tri-diagonalized matrix. For certain specialized purposes, other eigenvalue/vector methods (Jacobi; power method) are employed. In processing raw data to compute univariate moment statistics, Explorer uses procedures recommended by Spicer (1972) to insure accuracy and to circumvent round-off errors. Derflinger's (1979) general factor analysis computing algorithm (a Newton-Raphson method) was adapted for ML extraction. Documentation of additional sources for extraction and rotation procedures are referenced in the appropriate sections of the manual. Percentage points for the normal distribution uses the algorithm of Odeh and Evans (1974). With that exception all routines were coded by the author.

Explorer in Action: A Complete Example

The Data and Analysis

The data analyzed here are scores on 11 subtests of the Wechsler Intelligence Scales for Children, Revised (WISC-R; Wechsler, 1974). The children, who were leaning disabled, were enrolled in the California Center for Educational Therapy. A thorough description of the sample may be found in Tabachnick and Fidell (2007, Appendix B.3, pp. 938-939). Prior results with similar clinical samples (McMahon & Kunce, 1981; Naglieri, 1981; Peterson & Hart, 1979) have shown two clear factors—verbal comprehension (VC) and perceptual orientation (PO). A third factor, sometimes called "freedom from distractibility" (FD), also present, tends to be less well defined. However, the Coding subtest usually has the strongest saturation on FD, and sometimes Arithmetic and Digit Span load as well. With this in mind, the hypercon confirmatory rotation (Fleming, 2012) was used to verify this structure.

[4] This section is technical and may be omitted by most readers.

There are several hyperplane fitting rotations available in Explorer (Table 8), including hypercon. The Description column briefly summarizes each; for some empirical comparisons see Fleming (2012).

Table 8
Hyperplane Fitting Rotations in Explorer

Method	***Reference***	***Description***
Promax	Hendrickson & White (1964)	A good standby rotation—but different values of the power parameter should be tried, as well as weighted promax for complex solutions.
Promax target	(As above)	A useful confirmation rotation.
Promaj	Trendafilov (1994)	Purportedly an improvement over promax that eliminates subjectivity and replaces approximate with exact zeros in the target.
KC	Kaiser & Cerny (1979)	Works well with many conventional problems.
HH	Derflinger & Kaiser (1989)	Works in many situations where Casey fails. Both Casey and HH use varimax as a pre-rotation.
Hypermax	Fleming (2012)	Improves upon HH by using weighted varimax as pre-rotation; hence, works better when factors are very complex (variables load on more than one factor). But it can fail in rare circumstances.
Hyperalt	(This publication)	An experimental alternative to hypermax that uses quartimin rather than weighted varimax as the pre-rotation.
Hypercon	Fleming(2012)	A confirmatory hyperplane fitting rotation, in which the user pre-specifies hyperplane versus salient loadings.

The factor pattern for the initial solution (unweighted least-squares) can be seen on p. 16 of Appendix B. The hypercon solution is shown on p. 26. But for greater clarity the factor pattern and correlations are repeated in Table 9 (below).

The most salient loadings (those > 0.15 in absolute magnitude) are highlighted, so that the structure becomes clearer. In fact, all of the hypothesized hyperplane values for the hypercon solution are ≤ 0.15. Using this as a rough index for hyperplane count (i.e., values that are close to zero or essentially in the hyperplane), then, this count is as predicted, with 20 in the hyperplane and 13 salient loadings.

TABLE 9
Factor Pattern Matrix for
Hypercon Rotation of WISC-R Factors

	Subtest Factors			
Variable	VC	PO	FD	IFS
Information	***.79***	-.06	-.02	.99
Similarities	***.65***	.06	.04	.95
Arithmetic	**.60**	-.13	***.24***	.79
Vocabulary	***.64***	.03	.04	.93
Comprehension	***.81***	-.05	-.03	1.00
Digit Span	***.46***	-.07	***.32***	.57
Picture Completion	.02	***.61***	-.11	.96
Picture Arrangement	.07	***.37***	.08	.88
Block Design	.06	***.64***	.08	.96
Object Assembly	-.15	***.79***	-.06	.94
Coding	-.03	.07	***.67***	.98
SFI	.99	.97	.69	
Factor inter-correlations:				
VC	1.00			
PO	.575	1.00		
FD	.049	.253	1.00	

Note: Loadings ≥ .15 in absolute value are highlighted. IFS = index of factor simplicity; SFI = scale fit index. Total IFS = .90; total SFI = .92.

Measures of the simplicity of structure include the following:

- Bentler's simplicity index, a function of all loadings (Bentler, 1977). A value of 0 indicates maximum complexity, with the highest value of 1 when all variables are of complexity one (perfect cluster structure). For these comparisons, BSI was .99 (p. 35 of output).
- Lorenzo-Seva's (2003) simplicity index. The extrema are the same for this measure (0 to 1) but values tend to appear lower than BSI. For this problems LSI = .51 for hypercon (p. 35). As might be expected with a moderately complex solution LSI is "midrange."
- Fleming's (2003) scale fit index computed for each factor also ranges from 0 – 1. It is a function of the ratio of the average squared salient loadings (in this case those above .15) to the non-salients. These are summarized at the foot of each factor, with overall IFS = .94. But closer inspection shows that IFS values for Factors 1 and 2 are high (.99 and .97), while the value for Factor 3 (.69) is much lower.
- Kaiser's (1976) index is computed for each row of the loading matrix. This is also

a 0 – 1 measure. Values of .90 and above are considered quite good. Here the IFS values for each factor are given on the right side of Table 9, with overall IFS = .94 (p. 33). But the value for the Digit Span subtest (.57) is noticeably lower, as this variable splits its loadings between Factors 1 and 3.

- Using .15 as a rough cutoff value for separating high and low loadings, 20 elements from the hypercon solution were less in absolute value than this criterion. The average (root-mean-square) of these hyperplane values was .076.
- Finally, the graph of sorted absolute loadings is shown in Figure 3 where it is contrasted with orthogonal varimax loadings. Notice how much cleaner the structure is for the hyperplane fitting routine. Especially note how the hyperplane loadings form an almost straight but gradual line for the least values, taking a jump only after 20 ticks on the ordinate, where the loadings become "practically significant" (above .15).

Overall, the results favor hypercon over other rotations that were tried (not shown due to space limitations.)

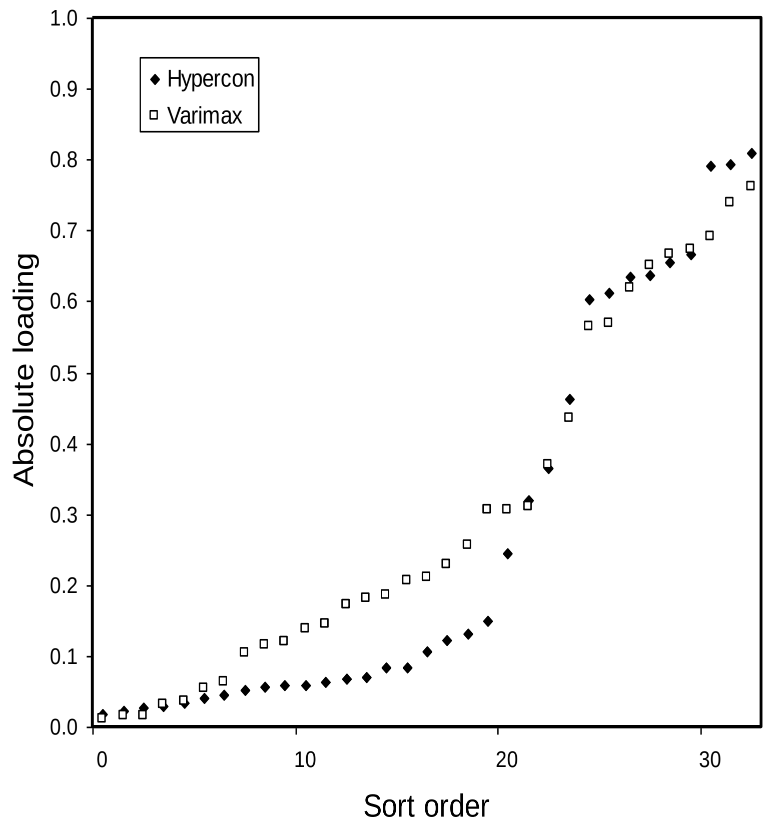

Figure 3: Sorted Absolute Loading Plot Comparing Hypercon and Varimax Rotations

Help Box 5:
Confirmatory Versus Exploratory Analysis: Some Practical Guidelines

Why would a confirmatory rotation be better than a good analytic rotation for certain problems? And if we are in confirmatory mode, why not use confirmatory factor analysis? These questions deserve further discussion.

As to the first question, remember that factor analysis is best applied to large samples. In the previous example the sample of 232 cases is “not bad” relative to the number of variables and factors, especially given that, with such clinical samples, it is difficult to find a large number of cases. But given the mixed results of prior research it might be concluded that this sample size isn’t big enough to “pin down” that elusive third factor with a conventional rotation, one that might capitalize on chance variations in the data in order to minimize the rotational discrepancy function *in the sample*. With the confirmatory rotation the researchers are saying, in effect, okay, this is what we think is the most likely outcome—let’s try it to see if it works. If not, we are not bound to stick with it, and can try other options.

As to the second issue, yes, confirmatory factor analysis is not an unreasonable option with this data. And there are, indeed, different schools of thought on this subject. This is not so much an either-or, which-is-really-best situation. Most interesting work in social science is complex, and is *almost never* purely exploratory or confirmatory, because the research questions allow for a little of both; so a “let’s find out” attitude calls for a flexible approach to methodology. Neither do results exist in isolation of human judgment; *it is never prudent to rely solely on mechanical or automated computer program defaults to make decisions for us*.

With these thoughts in mind, note that confirmatory factor analysis is scarcely ever *purely* confirmatory. Even when results appear to confirm basic hypotheses, at least some post hoc model modification is usually required in order to obtain an acceptable fit. Michael Browne (2001, p. 113) addresses this problem:

> Confirmatory factor analysis procedures are often used for exploratory purposes. Frequently a confirmatory factor analysis, with prespecified loadings, is rejected and a sequence of modifications is carried out in an attempt to improve fit. The procedure then becomes exploratory rather than confirmatory . . . In this situation the use of exploratory factor analysis, with rotation of the factor matrix, appears preferable.

Finally, note that one study in one sample is never completely sufficient to answer the research questions posed by any broad theory.

Second-order Factor Analysis of WISC-R Subtests
A second-order factor analysis was conducted based on the correlation matrix between the three first-order factors. The results are shown in the Appendix, pp. 43-54. The second-order loadings are:

	"General Intelligence" Loading	Communality
Factor 1: Verbal comprehension:	.569	.323
Factor 2: Perceptual orientation:	.988	.977
Factor 3: Freedom from distractibility:	.214	.046

Although Factor 2 has a much higher loading than Factor 1, this result may be partially artifactual, as the two first-order factors are substantially correlated (*r* = .575); and clear results in a factor analysis depend on having at least three good indicator variables for identifiability. (Second-order factor three has a low communality; hence it cannot be said to be a good indicator). Still, the results further illustrate Explorer's capabilities.

The Schmid-Leiman (1957) hierarchical factor structure is shown on p. 52. From this it can be seen that all of the variables except coding have at least a moderate loading on the general factor. But as for reliabilities (see Help Box 6) the higher-order coefficient omega (.581) is not particularly high, nor is the alpha coefficient (.575). Note also that the loadings for Factor 2 are all quite small, mainly because these items contribute significantly to the general factor.

Item Analysis Results for WISC-R Subtests
For test constructors Explorer has several item analysis features. (With the Wechsler tests, of course, the psychometric properties of the subtests are already well-known—we are not doing test development here, but nevertheless the various tables and statistics in the Appendix serves to illustrate the program's capabilities.)

First, the table of sorted loadings (p. 34 of Appendix) makes it easy to separate the salient items from the non-salients. (Note for Factor 3 there is really only one highly salient loading, and that is for the Coding subtest.)

Summary factor scale statistics (already discussed; see above) are shown on p. 35.

For factors with three or more salient loadings, separate factor analyses of these items are conducted, and from these, omega coefficients are computed. For Factors 1 and 2 these appear on pp. 36 and 37.

The correlations between the factor scales (scales based on the sums of the salient items) are given on p. 38. These will differ somewhat, of course, from the correlations among the latent variables themselves (p. 27).

Alpha coefficients for the factor scales are shown on p. 39, and item-total correlations, along with changes in alpha if item is deleted, are given on p. 40. In scale construction applications, items with low correlations, or which do not contribute to the overall reliability, are candidates for elimination.

Additional results from this analysis can be viewed in the Appendix.

Help Box 6:
The Alpha and Omega of Reliability

Factor analysis is often used to validate multidimensional scales of measurement. Factor scales have been used to measure, for example, dimensions of attitude, personality, and intellect, as well as many other attributes. In scale construction, many items are tried, and retained or eliminated based on psychometric evaluations, which include results of item analysis, item-total correlations, reliability, and factorial validity (i.e., items designed to measure a particular attribute should load uniquely on a given factor). As can be seen in the Explorer in Action example, Explorer provides many helpful statistics and measures to facilitate construction and evaluation of multidimensional factor scales.

Coefficient alpha (or Cronbach's alpha; Cronbach, 1951) is used extensively as a measure of internal consistency reliability, unidimensionality, and homogeneity, for scale items. But in fact, unless very strict assumptions are met (which is very often not the case), alpha is really none of these (Sijtmsa, 2009)—though it still can be useful as a *relative* measure for item analysis when creating or testing factor scales. Also, reporting alpha remains *de rigueur* for having articles on scale construction or test validation accepted for publication. Recent articles (see especially the series in V. 74, 2009, of *Psychometrika*) question this practice. Unfortunately, no uniform consensus exists at present as to which alternative measures should be used in place of alpha.

However, when using exploratory factor analysis, I concur with Revelle and Zinbarg (2009; also see Zinbarg & others, 2005), that McDonald's (1978; 1999) coefficient omega is a superior measure, and one that should be reported along with alpha in empirical studies (and eventually omega may supersede alpha).

One definition of alpha shows it to be a function of the ratio of the average inter-item covariance to the total variance of the items comprising the scale. McDonald (1978) showed, under fairly reasonable assumptions, that as the ratio of common factor variance to total item variance, coefficient omega is a more suitable measure of domain validity and generalizabilty.

When a single (general) factor is requested, Explorer computes both coefficients alpha and omega for the factor. When item analysis is requested in multiple factor solutions, Explorer computes these coefficients for each factor scale—a scale being defined by variable loadings greater than some minimal value (.40 is the default, but this value can be altered). It does this by performing *separate* factor analyses for each factor scale (i.e., it factors the items with the largest loadings).

In addition, when a second-order factor analysis is performed and a single, general second-order factor emerges, Explorer computes alpha and omega for this general factor. Revelle and others (2005) refer to this coefficient as ω_H (or higher-order omega).

McDonald (1978) also defined a coefficient that is equal to the ratio of the *total* common variance (based on both the second-order and lower order factors) to the total item variance. Explorer also outputs this ω_T statistic when a second-order analysis if performed with a single general factor.

For more details on these measures and their interpretation, refer to the articles cited in this section.

One note of caution is necessary: since alpha and omega are based on summated scales, items with negative loadings can result in "negative reliabilities," which of course are invalid; hence it may be necessary to recode variables in order to reverse the signs of certain items before performing the factor analysis.

Viewing and Printing Your Output on Microsoft Systems

Using Explorer in conjunction with Word or Notepad there are several options for nicely formatted output. (If you simply send your output file to your printer the formatting may overflow or otherwise not correctly fit the page.) A non-proportional font is required, with "Courier New" the default. As this typeface is a little faint, it may help to **bold** your output before printing.

1. Word 2010: "Everything fits on a page" (8.5 x 11 inches, per U. S. Standard). ***Do not use the ".txt" file extension with your output file*** (e.g., save as myoutput or results.out). Open your output file in Word, then select Page Layout → margins→ narrow. Note that the font type should be Courier New (a non-proportional font) 10-point. If this tends to look a bit faint it might look better if you first **bold** your document.

2. Notepad or other text processor: No pagination (saves paper).
Save your file with a "txt" extension (e.g., results.txt), and then open in Notepad. If darker type is wanted (optional) first **bold** your output before printing.

Other Computing Resources for Factor Analysis

SAS^{TM} , STATA^{TM}, and SPSS^{TM} are well-known statistical packages, but for factor analysis applications, other, free resources exist. The CEFA program (Browne and others, 2008) and Factor (Lorenzo-Seva & Ferrando, 2006), as with Explorer, provide features not found in these commercial packages, including newer and lesser known rotations. Gradient projection algorithms for factor rotation may be freely downloaded from http:/www.stat.ucla/research/GPA. These can be interfaced with SAS, SPSS, R language, etc. The M*plus* structural equation modeling program (Muthén & Muthén, 1998-2010) is not free but it includes facilities for *exploratory* SEM, which of course includes factor analysis.

References

Bartlett, M. S. (1950). Tests of significance in factor analysis. *British Journal of Psychology: Statistical Section*, *3*, 77-85.

Bentler, P. M. (1977). Factor simplicity index and transformations. *Psychometrika*, *42*, 277-295.

Bentler, P. M. (2007). Can scientifically useful hypotheses be tested with correlations? *American Psychologist*, *62*, 772-782.

Bernaards, C. I., & Jennrich, R. I. (2005). Gradient projection algorithms and software for arbitrary rotation criteria in factor analysis. *Educational and Psychological Measurement*, *65*, 676-696.

Browne, M. W. (1969). Fitting the factor analysis model. *Psychometrika*, *34*, 375-394.

Browne, M. W. (2001). An overview of analytic rotation in factor analysis. *Multivariate Behavioral Research*, *36*, 111-150.

Browne, M. W., Cudeck, R., Tateneni, K., & Mels, G. (2008). CEFA: Comprehensive Exploratory Factor Analysis, Version 3.03. [Online computer software and manual]. Retrieved from: www.http://faculty.psy.ohio-state.edu/edu/browne/.

Carroll, J. B. (1953). Approximating simple structure in factor analysis. *Psychometrika*, *18*, 23-28.

Clarkson, D. B., & Jennrich, R. I. (1988). Quartic rotation criteria and algorithms. *Psychometrika*, *53*, 251-259.

Crawford, C. B., & Ferguson, G. A. (1970). A general rotation criterion and its use in orthogonal rotation. *Psychometrika*, *35*, 321-332.

Cureton, E. E., & Mulaik, S. A. (1975). The weighted varimax rotation and the promax rotation. *Psychometrika*, *40*, 183-195.

Derflinger, G. (1979). A general computing algorithm for factor analysis. *Biometrics Journal*, *21*, 25-38.

Derflinger, G., & Kaiser, H. F. (1989). Method Horst-Hilsch (HH) for the transformation problem in exploratory factor analysis. *Psychological Reports*, *65*, 211-220.

Fleming, J. S. (2003). Computing measures of simplicity of fit for loadings in factor-analytically derived scales. *Behavior Research Methods, Instruments, and Computers*, *34*, 520-524.

Fleming, J. S. (2012). The case for hyperplane fitting rotations in factor analysis: A comparative study in simple structure. *Journal of Data Science*, *10*, 419-439.

Gorsuch, R. L. (1983). *Factor analysis* (2nd Ed.). Hillsdale, NJ: Erlbaum.

Guttman, L. (1944). General theory and methods for matric factoring. *Psychometrika*, *9*, 1-16.

Guttman, L. (1952). Multiple group methods for common factor analysis: Their basis, computation, and interpretation. *Psychometrika*, *17*, 209-222.

Guttman, L. (1953). Image theory for the structure of quantitative variates. *Psychometrika*, *18*, 277-296.

Harman, H. H. (1976). *Modern factor analysis* (3rd. Ed.). Chicago: University of Chicago Press.

Harman, H. H., & Jones, W. H. (1966). Factor analysis by minimizing residuals (minres). *Psychometrika*, *31*, 154-157.

Harris, C. W. (1962). Some Rao-Guttman relations. *Psychometrika*, *27*, 247-263.

Harris, C. W., & Kaiser, H. F. (1964). Oblique factor analytic solutions by orthogonal transformations. *Psychometrika*, *29*, 347-362.
Hendrickson, A. E., & White, P. O. (1964). Promax: A quick method for rotation to oblique simple structure. *British Journal of Statistical Psychology*, *17*, 65-70.
Holzinger, K. J. (1944). A simple method of factor analysis. *Psychometrika*, *9*, 257-262.
Horst, P. (1941). A non-graphical method for transforming an arbitrary factor matrix into a simple structure factor matrix. *Psychometrika*, *6*, 79-99.
Hotelling, H. (1933). Analysis of a complex of statistical variables into principal components. *Journal of Educational Psychology*, *24*, 498-520.
Jennrich, R.. I. (2001). A simple general procedure for orthogonal rotation. *Psychometrika*, *66*, 7-20.
Jennrich, R. I. (2002). A simple general procedure for oblique rotation. *Psychometrika*, *67*, 7-19.
Jennrich, R. I. (2004). Rotations to simple loadings using component loss functions. *Psychometrika*, *69*, 257-273.
Jennrich, R. I., & Bentler, P. M. (2011). Exploratory bi-factor analysis. *Psychometrika*, *76*, 537-549.
Jennrich, R. I., & Bentler, P. M. (2012). Exploratory bi-factor analysis: The oblique case. Psychometrika, *77*, 442-454.
Jennrich, R. I., & Sampson, P. F. (1966). Rotation for simple loadings. *Psychometrika*, *31*, 313-323.
Jöreskog, K. G. (1962). On the statistical treatment of residuals in factor analysis. *Psychometrika*, *27*, 335-354.
Jöreskog, K. G. (1979). Basic ideas of factor and component analysis. In K.G. Jöreskog & D. Sörbom (Eds.), *Advances in Factor Analysis and Structural Equation Models*. Uppsala, Sweden: Uppsala University.
Jöreskog, K. G. (2007). Factor analysis and its extensions. In R. Cudeck & R. C. MacCallum (Eds.), *Factor analysis at 100: Historical developments and future directions* (pp. 47-77). Mahwah, NJ: Erlbaum.
Kaiser, H. F. (1958). The varimax criterion for analytic rotation in factor analysis. *Psychometrika*, *23*, 187-200.
Kaiser, H. F. (1976). An index of factor simplicity. *Psychometrika*, *39*, 31-35.
Kaiser, H. F. (1981). A revised measure of sampling adequacy for factor-analytic data matrices. *Educational and Psychological Measurement*, *41*, 379-381.
Kaiser, H. F., & Caffrey, J. (1965). Alpha factor analysis. *Psychometrika*, *30*, 1-14.
Kaiser, H. F., & Cerny, B. A. (1978). Casey's method for fitting hyperplanes from an intermediate orthomax solution. *Multivariate Behavioral Research*, *13*, 395-401.
Lee, H. B., & Comrey, A. L. (1992). *A first course in factor analysis* (2nd Ed.). Hillsdale, NJ: Erlbaum.
Lorenzo-Seva (2000). The weighted oblimin rotation. *Psychometrika*, *65*, 301-318.
Lorenzo-Seva, U. (2003). A factor simplicity index. *Psychometrika*, *68*, 49-60.
Lorenzo-Seva, U., & Ferrando, P. J. (2006). Factor: A computer program to fit the exploratory factor analysis model. *Behavior Research Methods, Instruments, and Computers*, *38*, 88-91.

MacCallum, R. C., Browne, M. W., & Cai, L. (2007). Factor analysis models as approximations. In R. Cudeck & R. C. MacCallum (Eds.). *Factor analysis at 100: Historical developments and future directions* (pp. 153-175). Mawah, NJ: Erlbaum.
Mardia, K. V. (1970). Measures of multivariate skewness and kurtosis with applications. *Biometrika*, *57*, 519-530.
McDonald, R. P. (1978). Generalizability in factorable domains: "Domain validity and generalizability." *Educational and Psychological Measurement*, *38,* 75-79.
McDonald, R. P. (1985). *Test theory: A unified treatment*. Mahwah, NJ: Erlbaum.
McMahon, R. C., & Kunce, J. T. (1981). A comparison of the factor structure of the WISC and WISC-R in normal and exceptional groups. *Journal of Clinical Psychology*, *37*, 408-410.
Muthén, L. K.., & Muthén, B. O. (1998-2010). *Mplus user's guide (6th Ed)*. Los Angeles: Author.
Naglieri, J. A. (1981). Factor structure of the WISC-R for children identified as learning disabled. *Psychological Reports*, *49,* 891-895.
Neuhaus, J. O., & Wrigley, C. (1954). The quartimax method: An analytic approach to orthogonal simple structure. *British Journal of Statistical Psychology*, *7*, 81-91.
Odeh, R. E., & Evans, J. O. (1974). Algorithm AS 70: Percentage points of the normal distribution. *Applied Statistics, 23*, 96-97.
Peterson, C. R., & Hart, D. H. (1979). Factor structure of the WISC-R for a clinic-referred population and specific subgroups. *Journal of Consulting and Clinical Psychology*, *47*, 643-645.
Revelle, W., & Zinbarg, R. E. (2009). Coefficients alpha, beta, omega, and the GLB : Comments on Sijtsma. Psychometrika, 74, 145-154.
Rosenberg, M. (1965). *Society and the adolescent self-image*. Princeton, NJ: Princeton University Press.
Saunders, D. R. (1953). *An analytic method for rotation to orthogonal simple structure*. Research Bulletin 53-10. Princeton, NJ: Educational Testing Service.
Saunders, D. R. (1962). *Trans-varimax: Some properties of the ratiomax and equamax criteria for blind orthogonal rotation*. Paper presented at the annual meeting of the American Psychological Association, St. Louis, September, 1962.
Schmid, J., & Leiman, J, M. (1957). The development of hierarchical factor solutions. *Psychometrika*, *22*, 53-61.
Sijtsma, K. (2009). On the use, the misuse, and the very limited usefulness of coefficient alpha. *Psychometrika*, *74*, 107-120.
Stuive, I., Kiers, H. A. L., Timmerman, M. E., & ten Berge, J. M. F. (2008). The empirical verification of an assignment of items to subtests. *Educational and Psychological Measurement*, *68,* 923-939.
Tabachnick, B. G., & Fidell, L. S. (2007). *Using multivariate statistics* (5th Ed.). Boston, MA: Pearson Education, Inc.
Thurstone, L. L. (1947). *Multiple factor analysis*. Chicago: University of Chicago Press.
Trendavilov, N. (1994). A simple method for procrustean rotation in factor analysis using majorization theory. *Multivariate Behavioral Research*, *29,* 385-408.
Tucker, L. R. (1951). A method of synthesis of factor analysis studies. Personnel Research Section Report No. 984. Washington, D.C.: Department of the Army.

Wechsler, D. (1974). *Wechsler Intelligence Scale for Children – Revised*. New York: Psychological Corporation.

Widaman, K. F. (2007). Common factors versus components: Principals and principles, errors, and misconceptions. In R. Cudeck & R. C. MacCallum (Eds.). *Factor analysis at 100: Historical developments and future directions* (pp. 177-203). Mawah, NJ: Erlbaum.

Yates, A. (1987). Multivariate exploratory data analysis. Albany: SUNY Press.

Zhang, G. (2014). Estimating standard errors in factor analysis. *Multivariate Behavioral Research*, *49*, 339-352.

Zhang, G., Preacher, K. J., & Luo, S. (2010). Bootstrap confidence intervals for ordinary least squares factor loadings and correlations in exploratory factor analysis. *Multivariate Behavioral Research*, *45*, 104-134.

Zinbarg, R. E., Revelle, W., Yovel, I., & Li, W. (2005). Cronbach's α, Revelle's β, and McDonald's ω_H: Their relations with each other and two alternative conceptualizaions of reliability. *Psychometrika*, *70*, 123-133.

Zoski, K. W., & Jurs, S. (1996). An objective counterpart to the visual scree test for factor analysis: The standard error scree. *Educational and Psychological Measurement*, *56*, 443-4

Appendix A

Non-Parametric Bootstrap Standard Errors and Confidence Intervals for Loadings and Factor Correlations

Bootstrap standard errors (BSEs) for factor loadings and factor correlations, along with confidence intervals, are available in Explorer 4.0, but there are some restrictions:

1. Raw data, with no missing data, must be input. The <data> paragraph must specify **matrix = raw;**
2. The analysis paragraph must specify **boots =** *numboots***;** 500 is typical.
3. Only the correlation matrix may be factored (not the covariance matrix).
4. Variable and case selection are not allowed.
5. Communality estimates are based on squared multiple correlations.
6. Methods of extraction are currently limited to OLS, Alpha, ML, and iterated PFA.
7. Methods of rotation are currently restricted: (a) all orthogonal rotations may be used, but (b) only the following oblique rotations are available: direct quartimin (aka CF quartimax), oblique (Crawford-Ferguson) varimax, geomin, oblisim, bi-factor quartimin, and bi-factor geomin.

The method follows from Zhang (2014) and Zhang, Preacher, and Luo (2010). Although originally developed for ordinary least-squares factor analysis, the method is non-parametric and may be used with other methods as well, and these "…are consistent regardless of data distributions and model misspecification" (Zhang, 2014, p. 343).

For the "alignment problem," bootstrapped samples are matched to the original loading matrix using reflections of signs and column permutation, as described below.

Warning: Alignment can fail when samples are small (boostrapping requires at least "moderate" sample sizes), when too many or too few factors are extracted, or when the factor structure is unclear. On the positive side, BSEs work well with most "reasonable" data found in practice when the model[5] is correctly specified (the number of factors in particular).

Misalignment Problems and Heywood Cases

In order to insure accuracy the rotated bootstrapped factor loading and correlation matrices must be made to "match" the original set. This may require column interchange and/or reflection of signs. Here is how Explorer handles the alignment problem:

1. A matrix of congruence coefficients (or "unadjusted correlations," Tucker, 1951) is computed between the original and bootstrap sample loadings.
2. Based on the sizes of the coefficients in this matrix, a signed permutation matrix is formed to transform the bootstrapped loadings so that they are best matched to the

[5] Not everyone seems aware that EFA (as compared to, say, component analysis) is properly a statistical model subject to testing (Browne, 1969; Jöreskog, 1979).

original loading matrix in terms of factor ordering and signs. Factor intercorrelations need to be transformed as well.

3. If factors cannot be matched based by congruence coefficient (e.g., when two bootstrapped factors appear to match a single factor of the original matrix) the iteration is skipped. These are totaled and then reported at the end of the run.

Standard Errors When the Model is Misspecified

My belief is that when aligning factors, there should be a match in which corresponding bootstrap factors resemble the original factors, in the sense that, following transformation via a signed permutation matrix, a matrix of congruence coefficients should exhibit larger values on the diagonal than any off-diagonal elements in the same row or column. Departures may occur when, for example, one factor coalesces with another because the number of factors is incorrectly specified. This could indicate that the number factors should be reduced. Explorer tallies but *does not include* such cases in computing standard errors, and, if the percentage of such cases is large (e.g., > 50%) the user should take note.

When a Heywood case occurs (estimated unique variance less than zero) an attempt is made to "repair" the problem and the iterations halted. Repairable Heywood cases *are included* in Explorer, and they are tallied. Heywood cases could also represent model misspecification, usually due to lack of statistical power (small *N*), or from the failure to specify one or more additional factors. (As was stressed earlier, correctly specifying the number of factors is probably the most crucial decision when conducting EFA.)

Statistical Significance Criteria

Confidence intervals are based on alpha = .05, two-tailed, with Bonferroni adjustments for family-wise control of Type I errors.

Why Explorer Does Not Base Factor "Scales" on Significant Loadings

Explorer can compute factor scales based on highest loadings on each factor, which include item analysis and reliability coefficients. The criterion for a high loading may be adjusted by the user. So, why are the items included on each scale not based on the criterion of statistical significance? That is because as all researchers know, statistical significance does not necessarily imply "practical importance." With BSEs small appearing loadings (e.g., < .15) are sometimes significant. Of course factor scales may require adjustment by the user in cases where "high" loadings are not statistically significant.

Example: Two Factors for the Rosenberg Self-Esteem Scale

Data for 210 colleges students on the RSSE (Rosenberg, 1965) were previously collected by the author. Even though the total score is used as a measure of general self-esteem, the test is known to produce two factors, which were extracted here using OLS and rotated via quartimin. The tables from the output are reproduced below for 200 bootstrap samples. Asterisks mark the significant parameters. Notice that in this example, at least, had the conventional rule of loadings > .30 been applied, the same loadings would have met the criteria. (But for a counterexample, see Zhang, 2014, Table 3, p. 10.)

Q u a r t i m i n   F a c t o r   P a t t e r n

		F 1	F 2
1	Var 1	1.000*	-0.107
2	Var 2	1.016*	-0.130
3	Var 3	0.678*	0.221
4	Var 4	0.744*	-0.003
5	Var 5	0.558*	0.209
6	Var 6	0.824*	0.112
7	Var 7	0.765*	0.118
8	Var 8	0.126	0.571*
9	Var 9	-0.085	0.916*
10	Var 10	0.262	0.631*

F a c t o r   C o r r e l a t i o n s

	F 1	F 2
F 1	1.000	
F 2	0.675*	1.000

S t a n d a r d   E r r o r s   o f   L o a d i n g s

		F 1	F 2
1	Var 1	0.100	0.068
2	Var 2	0.094	0.055
3	Var 3	0.169	0.144
4	Var 4	0.095	0.079
5	Var 5	0.166	0.142
6	Var 6	0.106	0.082
7	Var 7	0.118	0.097
8	Var 8	0.172	0.174
9	Var 9	0.215	0.237
10	Var 10	0.200	0.188

```
Standard Error Summary Statistics:
----------------------------------

Number of samples          =     500
Number of Heywood Cases    =       0
Non-fixable Heywood Cases  =       0
Number of mis-matches      =      28
Effective number samples   =     472
Percentage of effective    =      94%

Average SE                 =  0.13495
```

(A mis-match occurs when the bootstrapped loadings cannot be aligned with the original loadings.)

```
Bonferroni corrected Z-scores for 95% confidence interval:
For loadings,     Z =      2.773
For correlations, Z =      1.960
```

S t a n d a r d   E r r o r s   o f   C o r r e l a t i o n s

	F 1	F 2
F 1	0.000	
F 2	0.179	0.000

Confidence Intervals

for Factor Loadings

	F 1	F 2
1	(0.724, 1.277)	(-0.296, 0.083)
2	(0.755, 1.278)	(-0.282, 0.022)
3	(0.210, 1.145)	(-0.179, 0.620)
4	(0.481, 1.007)	(-0.221, 0.215)
5	(0.099, 1.017)	(-0.186, 0.604)
6	(0.531, 1.117)	(-0.115, 0.338)
7	(0.437, 1.093)	(-0.151, 0.388)
8	(-0.350, 0.602)	(0.089, 1.053)
9	(-0.681, 0.511)	(0.260, 1.572)
10	(-0.294, 0.818)	(0.111, 1.152)

Confidence Intervals

for Factor Correlations

	F 1	F 2
F 1	(0.000, 0.000)	
F 2	(0.307, 0.867)	(0.000, 0.000)

Appendix B

Output from Hypercon Confirmatory Rotation for WISC-R Factors

[The actual output follows in the later pages of the manual (downloadable as a separate document). A brief, page by page, description appears below.]

Page(s)	Description
1	Echo back of input command file is printed. The time, date, and name of command and output files are listed first. The <data> paragraph specifies the number of cases, the format for reading the raw data, and the name of the file, which resides in the current directory. The <headers> and <labels> paragraphs provide run titles and variable labels, respectively. In the <analysis> paragraph the method of extraction (minres) and oblique rotation (hypercon) are specified. The number of factors is pre-specified (3) and the initial communalities are specified to be squared multiple correlations. The file for reading the zero-one hypercon weight matrix is also specified. A second-order factor analysis is requested, with one higher-order factor. Plots of the first 3 factors are specified in the <plots> paragraph, and maximal output s requested in the <output> paragraph.
2-3	Univariate summary statistics are given (only when raw data are input).
4	Most extreme cases are listed so that potential univariate outliers may be detected.
5	Correlation matrix is printed.
6	The eigenvalues of the correlation matrix are shown, along with percentages of variance.
7	Plot of the number eigenvalues against the number of factors (scree plot).
8	Multivariate summary statistics.
9	Most extreme cases, based on multivariate statistics (useful for detecting multivariate outliers).

10	Matrix of partial correlations (with multiple correlations on the diagonal). Many statistics useful to factor analysts are based on this matrix.
11	Measures of sampling adequacy. For an interpretation, see Kaiser (1981), who preferred these to the Kaiser-Meyer-Olkin statistic (also given here).
12	Minres (minimum residuals analysis) is the method of factoring. This page shows how many iterations it took (28) to find the solution, and how the convergence criterion changed with each iteration. If too many iterations occur it may be a sign that the starting values (initial communality estimates) are poor. (Maximum number of iterations is 200, by default.)
13	Final eigenvalues, of the "reduced" correlation matrix (correlation matrix with communalities inserted in the diagonal, or more technically, the covariance matrix of the common parts of the variables).
14	Initial and final communalities and unique variances. In this case, initial communalities were squared-multiple correlations of each variable with all of the others. Final communalities are found after the 28 iterations.
15	The fit measures here are chi-square and the corresponding normal curve values. Statistically speaking, a non-significant chi-square indicates a good fit, meaning that the number of factors is sufficient to account for the results.
16	Unrotated loadings are of less interest (are less interpretable) than the final, rotated loadings (elements of the pattern matrix).
17	Residual correlations are based on standardized residuals. These are, in other words, what remains of the correlation matrix after factors have been extracted.
18-19	The plot of standardize residuals and the resulting statistics below the graph give some indication of the effectiveness of the factoring. If values are large (p. 20) then the factors have not completely exhausted the information in the correlation matrix.
20	The factor score weights are the coefficients used to estimate the factor scores from the raw data. These are mainly of mathematical interest and tend to be difficult to interpret (they don't exhibit the same simple structure as the pattern matrix).
21	Covariances and correlations of score estimates. What to look for are large values in the diagonal. These are squared-multiple correlations of the variables with the factors. Note that the correlations of the *factor scores* are not generally quite the same as the correlations among the *factors*. This is a technical point, and it is related to the problem of factor indeterminacy, so the reader is referred to Mulaik (2010).

22 This page gives details on the type of oblique rotation that was performed (hypercon), and related information.

23 The (Cureton-Mulaik) row normalized loadings of the initial factor matrix that is then rotated by hypercon.

24 Target matrix. The zeros indicate items targeted for the hyperplane (the low loadings) and the ones indicate the items expected to have non-trivial loadings in the final solution.

25 Oblique transformation matrix. The final oblique solution is equal (in matrix algebra terms) to the initial solution post-multiplies by this transformation matrix.

26 Hypercon pattern matrix. This is the most important l result!

27 Factor intercorrelations are also an important part of the solution. If nothing else, report the pattern matrix and this matrix in your research report.

28 Factor structure. This is the matrix of correlations between the variables and the factors (see discussio
Refe n of interpretation of the matrices of factor analysis in the main text).
rence structure. This matrix is proportional to the factor pattern, but contains
29 semipartial correlations (Also see discussion in text.

30-32 Factor loading plots help to see visual the relationships among the factors. Values are from the reference structure matrix (proportional to the factor pattern, but scaled so that the largest possible values are ± 1.0).

33 Kaiser's indexes of factor simplicity for each variable.

34 Sorted loadings. Helps to see what variables are really most salient for each factor.

35 Factor scale summary statistics. Of most interest are the scale fit index and the Bentler and Lorenzo-Seva measures. See Fleming (2003) and Lorenzo-Seva (2003) for interpreting these – but in general, higher values indicate simpler structure.

36-37 Factor analysis of scale items. Items loading highest on each factor are separately factor analyzed,, and McDonald's coefficient omega is computer for each scale.

38 Factor scale intercorrelations. The correlations among the scales formed by the summation of variables with the highest loadings on each factor are presented

	her. (They differ from the correlations among the factors, discussed previously, on p. 28.)
39	Coefficients alpha for factor scales.
40-41	Item analysis. These statistics, along with the previous (p. 35), are of special interest to researchers interested in scale development, in which each factor purportedly represents a different construct. In such cases the researcher hopes that the factors will also be of low complexity. Item-total correlations and coefficient alpha for each of the factor scales are therefore useful information.
42	Factor score weights for the rotated factors.
43	Covariance of factor scores. Similar to for p. 21, except that these apply to the final oblique factors.
44	Details are given for the requested second-order factor analysis.
45	Eigenvalues for the first-order correlation matrix are given.
46	A scree plot of the eigenvalues based on the first-order factor correlations is shown.
47	The initial eigenvalues for the “reduced” correlation matrix are listed for the second-order solution.
48	Details of the factor extraction show that 13 iterations were required.
49	Final eigenvalues of the correlation matrix.
50	Second-order factor loadings.
51	Initial and final communalities and unique variances.
52	This is the Schmidt-Leiman hierarchical factor solution, exhibiting loadings on the first general factor (G1) and the residual, orthogonalized loadings on the three first-order factors (F1 – F3). Notice that the loadings for factor F2 are nearly zero because these are essentially “absorbed” by the G1 factor. This can also be seen from the second-order loading on p. 50, where the loading of F1 is .994. Reliabilities (omega and alpha) are given at the foot of this table.
53	Residual correlation matrix.
54-55	Plot of the standardized residuals, and table of largest residuals.

(The rest of the Appendix consists of the computer output, downloadable as a separate attachment.)